The Islamia University Of Bahawalpur,
Department of Computer Science & IT
Bahawalnagar Campus
Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Topic: Polynomial Approximations

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Polynomial Approximations:-

⇒ Polynomials:-

Introduction:-

Polynomials are functions with useful properties. Their relatively simple form makes them an ideal candidate to use as approximations for more complex functions.

برائے نام - فرضی/نقلی/باز متعدد، کئی، ایک سے زیادہ → Assumptive

⇒ Polynomials:- A polynomials in 'x' is a function of the form

$$p(x) = a_0 + a_1x + a_2x^2 + \cdots\cdots + a_nx^n$$

when $a_n \neq 0$; n is a non-negative integer.

And $a_0, a_1, a_2, \ldots\ldots, a_n$ are constant.

We say that this polynomial 'p' has degree equal to 'n'.

Degree of a Polynomial:- The degree of a polynomial is the highest power to which the argument, Here it is x, is raised power.

Characteristics ~~or Versatil~~ of polynomials

→ These are relatively simple functions to deal with.
For exp. They are easy to differentiate and integrate.
→ The function of interest can be easily approximated by a polynomial.

⇔ Exp's ⇔

⇒

(a) $f(x) = x^2 - 2 - \frac{1}{x}$ (b) $f(x) = x^4 + x - 6$

(c) $f(x) = 1$

(d) $f(x) = mx + c$, m & c are consts.

(e) $f(x) = 1 - x^6 + 3x^3 - 5x^2$ ⇒ yes. deg-6

(f) $f(x) = 2 + 3e^x - 4e^{2x}$

(g) $f(x) = \cos x + \sin^2 x$

(a) Not a polynomial. bcz. $\frac{1}{x}$ term
-ve power argument not allowed.

(b) is polynomial of degree '6'

(c) is polynomial of degree zero

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⇒ What about 'f' & g expls.?
exp 'f' & 'g' are not polynomials.
Be.g. There Maclaurin expansion have infinitely many terms.
⇒ There are many ways for polynomial (⇒ approximation) The Ist one is

① Taylor series:-

Taylor series is a useful/easy way of approximating functions by polynomials.
The Taylor series expansion of a function $f(x)$ about $x = a$ may be stated as

$$f(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \frac{1}{3!}(x-a)^3 f'''(a) + \cdots$$

Consider only points 'x' which are near to 'a', when using this formula.

** The general idea when using this formula in practise is to consider only points 'x' which are near to 'a'. **

→ As $x-a$ will be small
Then $(x-a)^2$ will be even smaller
$(x-a)^3$ will be very smaller still
and so on.

~~This~~ or above property
This property gives us confidence to simply neglect the terms beyond a certain power, or ~~to~~ ~~put~~ another way, to 'truncate' the series.

Exp:- Find the Taylor polynomial of degree '2' about the point $x = 1$ for the function
$$f(x) = \ln(x)$$

The Taylor series expansion

$$f(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots$$

Here $a = 1$

Now
$$f(a) = \ln(a)$$
$$= \ln(1)$$
$$f(a) = 0$$

$$f'(x) = \frac{1}{x} \qquad f'(a) = \frac{1}{a}$$

at $a = 1$
$$f'(a) = \frac{1}{1}$$
$$f'(a) = 1$$

Similarly
$$f''(a) = -\frac{1}{a^2} = \frac{-1}{(1)^2} = -1$$

$$f'''(a) = \frac{2}{a^3} = \frac{2}{(1)^3} = 2$$

Hence $f(x) = f(a) + (x-a) f'(a) + \frac{1}{2}(x-a)^2 f''(a) + \cdots$
$$\ln(x) \approx 0 + (x-1) - \frac{1}{2}(x-1)^2$$

Example of the Polynomial Approximation (Tayler Series Method).

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$$\ln(x) = x - 1 - \frac{1}{2}(x^2 + 1 - 2x)$$
$$= x - 1 - \frac{1}{2}x^2 - \frac{1}{2} + x$$
$$\ln(x) = -\frac{3}{2} + 2x - \frac{1}{2}x^2$$

Which is reasonably accurate for x close to '1'.
You can check on a calculator or compute
For exp x b/w 0.9 to 1.1
The polynomial & logarithm agree to at least '3' decimal places.

Draw back of Taylor Series approximation.
→ With this approach, we need to find possibly many derivatives of 'f'.
→ There is some doubt over what is the best choice of 'a'.
→ It ~~is~~ ~~has~~ can sometimes have limitted appeal as a means

Of approximating functions by polynomials.

Best of Luck